

### 11. Условия типового расчета

**Задачи 1, 3.** Найти общее решение дифференциального уравнения.

**Задачи 2, 4.** Найти частное решение дифференциального уравнения, удовлетворяющее заданным начальным условиям.

**Задача 5.** Найти общее решение линейного неоднородного дифференциального уравнения второго порядка, если известно одно частное решение  $y_1(x)$  соответствующего однородного уравнения.

**Задача 6.** Методом изоклин построить приближенно интегральные кривые дифференциального уравнения первого порядка.

Таблица 2.

№ вар.	Задача 1.	№ вар.	Задача 1.
1	$y''(y+1)^2 = (y'+1)^2 y'$	16	$y'' = \frac{1}{y^3}$
2	$(1+y^2)y'' - 2y(y')^2 = 0$	17	$xy'' - y' = xy'$
3	$(1+x^2)y'' + 1 + (y')^2 = 0$	18	$(x-2)^2 y'' = (y'+2)^2$
4	$y''(\sin x + 1) - y' \cos x = 0$	19	$y'' \operatorname{tg} x - 2y' - 4 = 0$
5	$(x-1)^2 y'' = (y'+3)^2$	20	$yy'' - 2(y')^2 + y^3 y' = 0$
6	$x^2 y'' + xy' = 4x + 1$	21	$y'' = 2(y')^2 \operatorname{ctg} y$
7	$xy'' = y' \ln \frac{y'}{x}$	22	$y''(\sin x + \cos x) - y'(\cos x - \sin x) = 0$
8	$yy'' - (y')^2 + y^2 y' = 0$	23	$xy'' + y' - x = 0$
9	$y^2 y'' - 2y(y')^2 + (y')^3 = 0$	24	$y''(\ln x - 1) = \frac{y'}{x}$
10	$x^2 y'' + xy' = 1$	25	$y'' - y' \operatorname{tg} x - 1 = 0$
11	$y''(y+5) - (y')^2 = 0$	26	$y''(x^3 - 5) = 3x^2 y'$
12	$y'' \sin^2 y - (y')^2 \sin 2y = 0$	27	$(x \ln x)y'' = (1 + \ln x)y'$
13	$y'' \sin 2x = 2y' + 4$	28	$yy'' + (y')^2 = 2y^2 (y')^2$
14	$yy'' - y'(1+y') = 0$	29	$yy'' = (y')^2 - \frac{y'}{y}$
15	$y'' + \frac{2}{1-y} (y')^2 = 0$	30	$(1 + e^{-x})y'' = y'$

№ варианта	Задача 2.
1	$y'' - \frac{y'}{x} - \frac{x^2}{y'} = 0, y(1) = \frac{4}{15}, y'(1) = 1$
2	$y'' - 2y' \cdot \operatorname{ctgx} - \sin^3 x = 0, y\left(\frac{\pi}{2}\right) = \frac{\pi}{4}, y'\left(\frac{\pi}{2}\right) = 1$
3	$2yy'' - (y')^2 - y^2 = 0, y(0) = 1, y'(0) = 1$
4	$y'' \sin y + (y')^2 \cos y - y' \sin y = 0, y(0) = \frac{\pi}{2}, y'(0) = 1$
5	$yy'' - 2yy' \ln y = (y')^2, y(-1) = e, y'(-1) = e$
6	$yy'' + (y')^2 + 1 = 0, y(1) = 1, y'(1) = 1$
7	$yy'' + (y')^2 = (y')^3, y(0) = 1, y'(0) = \frac{1}{2}$
8	$xy'' = y' - 5, y(1) = 0, y'(1) = 0$
9	$y'y''(1+x^2) = x(1+(y')^2), y(1) = \frac{1}{2}, y'(1) = 1$
10	$y^2y'' - y'(y'-1)^2 = 0, y(0) = -1, y'(0) = 2$
11	$y'' + y' \left( \operatorname{ctgx} - \frac{1}{x} \right) - x^2 \operatorname{ctgx} - x = 0, y(1) = 0, y'(1) = 1$
12	$xy'' - y' = x(\ln x - 1), y(1) = 1, y'(1) = 1$
13	$y'' - y'(y' + 2y - y^2) = 0, y(0) = 1, y'(0) = 1$
14	$(1+x^2)y'' + 2xy' = x^3, y(0) = 0, y'(0) = 1$
15	$xy'' - y' - x^2 = 0, y(1) = 1, y'(1) = -1$
16	$xy'' - (x+1)y' + e^x = 0, y(1) = e, y'(1) = 2e$
17	$y''(y+1)^2 = (y')^3, y(1) = 0, y'(1) = \frac{1}{2}$
18	$yy'' - (y')^2 + (y')^3 = 0, y(0) = 1, y'(0) = \frac{1}{2}$
19	$yy'' - (y')^2 + y^2y' = 0, y(0) = 1, y'(0) = 1$
20	$(x+1)y'' = y' + 1, y(1) = -1, y'(1) = 1$
21	$xy'' - y' - x^2 \cos x = 0, y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{8}, y'\left(\frac{\pi}{2}\right) = \pi$
22	$y'' + y'(y^2 + 2y + y') = 0, y(0) = \frac{1}{2}, y'(0) = -\frac{1}{4}$
23	$y''(y-4)^2 = y'(y'-4)^2, y(1) = 5, y'(1) = \frac{9}{2}$
24	$yy'' + (y')^2 - 2yy' = 0, y(0) = 1, y'(0) = 2$
25	$y'' + \frac{4}{2-y}(y')^2 = 0, y(2) = 1, y'(2) = 1$
26	$yy'' + (y')^2 + 2yy' = 0, y(0) = 2, y'(0) = 1$
27	$yy'' + 3(y')^2 + 4yy' = 0, y(1) = 1, y'(1) = 1$

28	$xy'' + 2y' = \frac{1}{x^2}, y(1) = 1, y'(1) = 1$
29	$(x^2 + 1)y'' = (y')^2 + 1, y(0) = 1, y'(0) = 1$
30	$yy'' + (y')^2 = 3(y')^3, y(0) = 1, y'(0) = \frac{1}{2}$

Таблица 4.

№ вар.	Задача 3.	№ вар.	Задача 3.
1	$2y'' + 5y' = x - \sin\left(\frac{5x}{2}\right) + e^{\frac{5x}{2}}$	16	$y'' - 2y' - 8y = 16e^{-2x} + 17 \sin x$
2	$y'' - 3y' + \frac{5}{2}y = 5x^2 + e^{\frac{3x}{2}}$	17	$y'' - 10y' + 25y = 4e^{5x}$
3	$9y'' - 6y' + y = \sin x - e^{\frac{x}{3}}$	18	$y'' - 4y' + 3y = e^x - e^{-x}$
4	$y'' + 8y' + 16y = \cos x + e^{-4x}$	19	$6y'' - y' - y = \sin \frac{x}{2} + x$
5	$2y'' - 5y' - 3y = 2e^{3x} + \sin \frac{x}{2}$	20	$9y'' - 6y' + y = 4 \cos \frac{x}{3} + 18e^{\frac{x}{3}}$
6	$y'' + 3y' - 4y = e^x - x$	21	$y'' - 4y' + 4y = 4x^3 - 2x + \cos x$
7	$2y'' - y' - y = e^x + \sin x$	22	$y'' - 4y' + 3y = 3 \sin 3x + e^x$
8	$3y'' - 5y' - 2y = e^{2x} + x^2$	23	$y'' + 2y' - 8y = 6e^{2x} - x^2$
9	$2y'' + 5y' - 3y = 6x^2 + x - e^{-3x}$	24	$y'' + 6y' + 13y = 4e^{-3x} + 3 \cos 2x$
10	$y'' + 3y' - 4y = e^{-4x} + e^{4x}$	25	$y'' - 3y' - 4y = 34 \sin 4x + 5e^{-x}$
11	$6y'' - y' - y = \cos \frac{x}{2} + e^{-\frac{x}{3}}$	26	$y'' + 6y' + 25y = 16e^{-3x} + 73 \cos 4x$
12	$9y'' - 6y' + y = 9e^{\frac{x}{3}}$	27	$y'' - 4y' + 4y = e^{2x} - 16x^2 + 4$
13	$y'' - 8y' + 16y = 4 \sin 4x + e^{4x}$	28	$y'' - 8y' + 25y = 4 \sin 3x + 9e^{4x}$
14	$5y'' - 2y' + y = \frac{2}{5} \cos \frac{x}{5}$	29	$5y'' - 6y' + 5y = 73 \cos \frac{4x}{5} - 1$
15	$y'' - 12y' + 36y = 72 \sin 6x + x$	30	$5y'' - 2y' + y = xe^{\frac{x}{5}}$

№ варианта	Задача 4.
1	$2y'' - 5y' - 3y = e^{3x}, y(0) = 0, y'(0) = \frac{8}{7}$
2	$4y'' + 4y' + y = 2e^{-\frac{x}{2}}, y(0) = 1, y'(0) = 1$
3	$2y'' - y' - y = e^x, y(0) = 3, y'(0) = \frac{1}{3}$
4	$y'' + y' = x + e^x, y(0) = \frac{1}{2}, y'(0) = \frac{1}{2}$
5	$y'' - 12y' + 36y = 36x + 2e^{6x}, y(0) = -\frac{2}{3}, y'(0) = 0$
6	$y'' - 10y' + 25y = 5 \sin 5x, y(0) = \frac{11}{10}, y'(0) = 0$
7	$3y'' - 2y' = x^2 + e^{-\frac{2x}{3}}, y(0) = \frac{3}{8}, y'(0) = 0$
8	$y'' - 8y' + 16y = 16 \cos 4x - 1, y(0) = -\frac{1}{16}, y'(0) = 0$
9	$y'' - 4y' + 4y = 8x - 4 \cos 2x, y(0) = 1, y'(0) = -1$
10	$y'' - 12y' + 36y = 18x^3 + 1, y(0) = \frac{1}{12}, y'(0) = -\frac{3}{4}$
11	$4y'' + 4y' + y = x^3 + 6x^2, y(0) = 2, y'(0) = 0$
12	$4y'' - 4y' + 2y = 5e^{\frac{x}{2}} - 4x, y(0) = 1, y'(0) = 1$
13	$y'' - 2y' + 5y = -5x^3 - 4x^2 + 2x, y(0) = -\frac{1}{5}, y'(0) = -3$
14	$2y'' + 5y' - 3y = e^{\frac{x}{2}} + 6x, y(0) = -\frac{1}{3}, y'(0) = \frac{1}{7}$
15	$6y'' - y' - y = e^{\frac{x}{2}} + e^{-\frac{x}{2}}, y(0) = 1, y'(0) = \frac{1}{5}$
16	$4y'' + 4y' + y = 2x^2 - 4, y(0) = 4, y'(0) = 0$
17	$y'' - 5y' + 6y = 26 \sin 2x + 1, y(0) = -\frac{1}{3}, y'(0) = -1$
18	$9y'' + 12y' + 4y = \cos \frac{2x}{3} - 8, y(0) = -2, y'(0) = 1$
19	$y'' - 8y' + 16y = 4e^{4x}, y(0) = 1, y'(0) = 1$
20	$y'' + 2y' + 5y = 5x^2 - x, y(0) = 1, y'(0) = 0$
21	$y'' - 5y' + 6y = 3e^{3x} + 1, y(0) = \frac{1}{6}, y'(0) = 0$
22	$4y'' + 4y' + y = 8e^{-\frac{x}{2}} + x, y(0) = 0, y'(0) = 1$
23	$y'' + 2y' + 5y = 4e^{-x} + \sin 2x, y(0) = \frac{13}{17}, y'(0) = \frac{2}{17}$
24	$y'' + 4y' + 4y = 2e^{-2x} + x, y(0) = \frac{3}{4}, y'(0) = \frac{1}{4}$
25	$y'' - 2y' + y = 4e^x + x^2 - 4x, y(0) = 2, y'(0) = -2$

26	$y'' + 12y' + 36y = 2e^{-6x} - 108x, y(0) = 0, y'(0) = 0$
27	$y'' - 6y' + 13y = 25 \sin 2x, y(0) = \frac{1}{3}, y'(0) = 1$
28	$y'' + 6y' + 9y = 2e^{-3x} - 9x, y(0) = \frac{2}{3}, y'(0) = 0$
29	$y'' + 3y' - 4y = e^{4x} - e^{-4x}, y(0) = \frac{1}{24}, y'(0) = -\frac{31}{30}$
30	$y'' - 4y = 4 \cos 2x + 4e^{2x}, y(0) = \frac{1}{2}, y'(0) = 1$

Таблица 6.

№ варианта	Задача 5.
1	$12x^2 y'' + 5xy' + y = \sqrt[3]{x} + \sqrt[4]{x}, y_1(x) = \sqrt[4]{x}$
2	$xy'' - (1 + 2x \operatorname{ctg} x) y' + (\operatorname{ctg} x + 2x \operatorname{ctg}^2 x + x) y = 2x^3 \sin x, y_1(x) = \sin x$
3	$x^2 y'' - 2x(x+1) y' + x(x+2) y = 6x^4 e^{3x}, y_1(x) = e^x$
4	$x^2 y'' - xy' + y = 6x \ln x, y_1(x) = x$
5	$y''(2x - x^2) + y'(2x - 2) - 2y = (x - 2)^2, y_1(x) = x^2$
6	$(x^2 \ln^2 x) y'' - (2x \ln x) y' + (\ln x + 2) y = \ln^4 x, y_1(x) = \ln x$
7	$(\operatorname{ctg} x) y'' + 2y' + (2 \operatorname{tg} x + \operatorname{ctg} x) y = \cos^2 x, y_1(x) = \cos x$
8	$x^2 y'' + xy' - y = 2 \ln x, y_1(x) = x$
9	$x^2 y'' - 2x^2 y' + (x^2 - 2) y = 3x^3 e^{2x}, y_1(x) = x^2 e^x$
10	$x^2 y'' - 5xy' + 8y = 2 \ln x, y_1(x) = x^2$
11	$x^2 y'' + 4xy' + 2y = \sin x, y_1(x) = \frac{1}{x}$
12	$xy'' + (2x \operatorname{tg} x - 1) y' + (x + 2x \operatorname{tg}^2 x - \operatorname{tg} x) y = 2x \cos x, y_1(x) = \cos x$
13	$xy'' - (2x - 2) y' + (x - 2) y = 4e^{3x}, y_1(x) = e^x$
14	$(x^2 \ln^2 x) y'' - 2x \ln x (\ln x + 1) y' + (2 + 3 \ln x + 2 \ln^2 x) y = \ln^3 x, y_1(x) = x \ln x$
15	$4x^2 y'' - 4xy' + 3y = \sqrt{x} \ln x, y_1(x) = \sqrt{x}$
16	$x^2 y'' - 4xy' + 6y = x^4 \sin x, y_1(x) = x^2$
17	$y'' \operatorname{tg} x - 2y' + y(2 \operatorname{ctg} x + \operatorname{tg} x) = \sin^2 x, y_1(x) = \sin x$
18	$(x^2 \ln^2 x) y'' - x \ln x (\ln x + 2) y' + 2(\ln x + 1) y = 2 \ln^3 x, y_1(x) = \ln x$
19	$xy'' + y'(3 - 2x) + y(x - 3) = 2, y_1(x) = e^x$
20	$x^2 y'' - xy' + y = x \ln x, y_1(x) = x$
21	$x^2 y'' - 4xy' + 6y = x^2 - 1, y_1(x) = x^3$
22	$xy'' - (2x + 1) y' + y(x + 1) = 2xe^x, y_1(x) = e^x$
23	$(x - 1) y'' - xy' + y = (x - 1)^2, y_1(x) = e^x$
24	$x^2 y'' - x(2 + x) y' + (2 + x) y = 3x^3, y_1(x) = x$

25	$x^2 y'' - 9xy' + 24y = 5x^4, y_1(x) = x^4$
26	$x^2 y'' - 7xy' + 15y = 5x^6(x-1), y_1(x) = x^3$
27	$x^2(\ln x + 1)y'' - x(2\ln x + 3)y' + (2\ln x + 3)y = 3x(\ln x + 1)^2, y_1(x) = x$
28	$y'' - (3\operatorname{ctg} x)y' + y(1 + 3\operatorname{ctg}^2 x) = \sin^3 x, y_1(x) = \sin x$
29	$x^2(1 - \ln x)y'' + xy' - y = x^2(1 - \ln x)^2, y_1(x) = \ln x$
30	$2x^2 y'' - 3xy' + 2y = 6x^3, y_1(x) = \sqrt{x}$

Таблица 7.

№ варианта	Задача 6.	№ варианта	Задача 6.
1	$y' = (y + e^x)^2$	16	$y' = \cos(y - x)$
2	$y' = (x + e^y)^2$	17	$y' = y^2 - 2y$
3	$y' = 2^{\frac{y}{x}}$	18	$y' = 2y - y^2$
4	$y' = \sqrt{y} - x$	19	$y' = \sqrt{y - x^2}$
5	$y' = \frac{1}{4}(x^3 + y)^2$	20	$y' = \sqrt{x^2 - y}$
6	$y' = -\frac{1}{4}(x - y^3)^2$	21	$y' = \sin(x + y)$
7	$y' = -\sqrt{y} - 2x$	22	$y' = \ln(x + y)$
8	$y' = x - 2\sqrt{y}$	23	$y' = y - e^{-x}$
9	$y' = \ln y - \ln x$	24	$y' = y - \frac{1}{2}e^x$
10	$y' = \left(y - \frac{1}{4}x\right)^2$	25	$y' = \frac{y + 4}{x^2 + 1}$
11	$y' = (2x - y)^3$	26	$y' = -\frac{y - 1}{x^2 + 1}$
12	$y' = -\ln(y - x)$	27	$y' = y + e^{-x}$
13	$y' = (y - 3x)^2$	28	$y' = \ln y + x$
14	$y' = \left(y + \frac{x}{2}\right)^3$	29	$y' = y + x^2$
15	$y' = \sqrt{(y - 2x)^3}$	30	$y' = y - x^2$